time change in the spacing $\lambda(t)$ between centers of two light bands, one of which was between the 6 and 7 and the other between the 8 and 9 cylinders (counting from the top), is shown by curve 1 in Fig. 6b.

Constants of the material in application to a two-element Voigt model were first obtained in creep tests for the computation: $C_1 = 7 \cdot 10^6$ N/m², $C_2 = 6 \cdot 10^5$ N/m², $\mu_1 = 1.5 \cdot 10^3$ N·sec/m², $\mu_2 = 2 \cdot 10^3$ N·sec/m². The results of the computation, performed on the effect of the above-mentioned nonstationary pulse (for $\sigma_+ = 0$, h = 1.5 cm) in the form of the function $\lambda(t)$, are shown by curve 2. The beginning of the slot formation in the computation and the tests agreed with high accuracy and corresponds to the value $t_k \approx 20$ msec.

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A POINT EXPLOSION IN A COMPRESSED MULTICOMPONENT MIXTURE

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UDC 534.222

Among the various approaches to the study of the motions of a compressed medium, a special place is occupied by self-similar methods for the solution of the hydrodynamic equations, making it possible to reduce the problem to the investigation of ordinary differential equations. In [1] a scheme was developed for the calculation of the self-similar motions of an ideal gas in an incompressible liquid in the case of a strong point explosion; in [2] the methods of [1] were generalized for the case of a strong explosion in a compressible medium. Both pieces of work considered one-component media. At the same time, the study of explosive motions in media consisting of several components is of considerable interest for practical purposes.

In order that the problem of a strong point explosion in a compressible medium be self-similar, it is sufficient that the equation of state of the medium have the form

$$\varepsilon(p, \rho) = \frac{p}{\rho_0} \Phi\left(\frac{\rho}{\rho_0}\right) + \text{const}, \tag{1}$$

where ε is the internal energy; p and ρ are the pressure and the density, respectively; ρ_0 is a constant with the dimensionality of density; and Φ is an arbitrary function of its argument. Directly from relationship (1) there follows the equation of the adiabat of the medium:

$$p(\rho) = \psi(S)\chi(\rho/\rho_0),$$

where S is the entropy. The connection between the functions Φ and χ is determined by the formulas

$$\Phi(R) = \frac{1}{\chi(R)}\left\{c + \int \frac{\chi(R)}{R^2}\, dR\right\}, \quad \chi(R) = \frac{c}{\Phi(R)} \exp \int \frac{dR}{R^2 \Phi(R)}. \tag{1a}$$

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In what follows, we shall limit ourselves to an equation of state of the form of (1), taking account of the multicomponent character of the medium. At the initial moment of time $t = 0$, at the origin of coordinates let the energy E_0 be evolved. As a result, a shock wave is propagated over a medium with an initial density ρ_1. We limit ourselves to the case of a strong explosion; we neglect the pressure ahead of the shock front. The problem is then self-similar, and all the quantities describing the motion of the medium will depend only on the one variable λ [1]:

$$\lambda = r/r_D, \quad r_D = (Et^2/\rho_0)^{1/5}, \quad E = E_0/\alpha, \tag{2}$$

where r_D is the radius of the shock front; t is the time; and α is a parameter, determined from the condition of the equality of the total energy of the perturbed substance to the energy of the explosion E_0.

Using the self-similarity of the problem, and introducing the dimensionless pressure, specific volume, and velocity using the formulas

$$v = av_1V(\lambda), \quad u = \frac{2}{3}\frac{r}{t}(1-a)U(\lambda),$$
$$p = \frac{4}{25}\frac{r^2}{t^2}\frac{1-a}{v_1}P(\lambda), \tag{3}$$

we can obtain a system of three equations for the functions $V(\lambda)$, $U(\lambda)$, and $P(\lambda)$. The quantity a in (3) is the degree of compression of the substance at the shock front; $v_1 = 1/\rho_1$.

For the subsequent calculations, a concrete form must be assigned to the function Φ. In accordance with [3], under the assumption of the equality of the pressures in all the components, the following equation is obtained for a multicomponent medium. For the specific volume and specific energy of a mixture of substances we have

$$v(p) = \sum_i R_i v_i(p), \quad E(p) = \sum_i R_i E_i(p), \tag{4}$$

where R_i is the weight content of the corresponding components. The sum in (4) is taken over all the components of the mixture. We shall assume that the concrete form of $v_i(p)$ is determined by the Tate equation [3]:

$$v_i(p) = v_{0i}\left[\frac{\gamma_i p}{\rho_{0i}c_{0i}^2} + 1\right]^{-\frac{1}{\gamma_i}}, \tag{5}$$

where c_{0i} is the speed of sound in the corresponding component; γ_i is the adiabatic index. As has been shown by a number of authors [3, 4], an equation of state of this kind satisfactorily describes the behavior of such media as clay, water-saturated sand, etc., up to pressures on the order of a few kilobars. Taking account of (5) and (1a), for the internal energy of a multicomponent medium E(p) the following expressions can be obtained:

$$E = \frac{v(q)}{q}\left[\frac{\varphi(q)}{B} - q\right],$$
$$\varphi(q) = \frac{B}{v(q)}\left\{A + \frac{1}{v_0}\sum_i \frac{\gamma_i R_i v_{0i} B_i}{(\gamma_i - 1)}\left(\frac{Bq}{B_i} + 1\right)^{\frac{\gamma_i - 1}{\gamma_i}}\right\},$$
$$A = -\frac{1}{v_0}\sum_i \frac{\gamma_i R_i v_{0i} B_i}{(\gamma_i - 1)}, \quad B = \sum_i R_i B_i,$$
$$B_i = \frac{\rho_{0i}c_{0i}^2}{\gamma_i}, \quad q = \frac{p}{B}, \quad a_1 v = f(q),$$
$$\Phi(a_1 v) = \frac{f(q)}{q}\left[\frac{\varphi(q)}{B} - q\right], \quad a_1 = a\frac{v_1}{v_0},$$
$$v_0 = \frac{1}{\rho_0}. \tag{6}$$

Following the method of [2] and taking account of (6), after certain transformations, the system of three equations for $V(\lambda)$, $U(\lambda)$, and $P(\lambda)$ can be written in the form

$$\frac{dq}{dU} = (1-a)\frac{f\left[\frac{\varphi}{B} - q\right]}{[1-(1-a)U]}\,\frac{Uq(1-a)[2(1-a)U+1] + \frac{1}{2}[1-(1-a)U][1-4(1-a)U]\left[\frac{\varphi}{B} - q\right]}{\frac{3}{2B}U^2(1-a)^2[f'\varphi q + f\varphi' q - f\varphi] + (1-a)qf'U[(1-a)U-4]\left[\frac{\varphi}{B} - q\right] + f'\left[\frac{5}{2} - (1-a)U\right][1-(1-a)U]\left[\frac{\varphi}{B} - q\right]^2}. \tag{7}$$

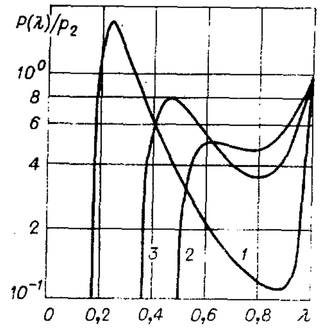

Fig. 1

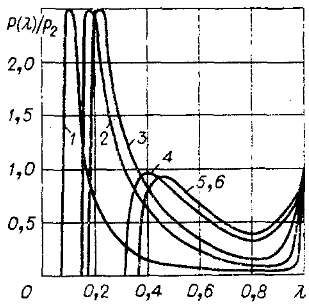

Fig. 2

Primes denote differentiation with respect to the argument q. The initial condition for Eq. (7) is

$$q(1) = f^{-1}(a_1), \tag{8}$$

where f^{-1} is a function inverse to f. The initial pressure q(1) corresponds to the pressure q at the shock front.

Equation (7) with the initial condition (8) was integrated by the Runge–Kutta method.

For a transition to dimensional variables, we need to know the value of α in formulas (2). The law of conservation of energy, written using (3), gives an expression for the parameter α

$$\alpha = \frac{16\pi}{25}\frac{(1-a)}{aV_1}\int_{\lambda^*}^{1}\frac{\lambda^4 d\lambda}{V(\lambda)}\left[\frac{U^2(\lambda)}{2} + P(\lambda)\frac{\Phi(a_1V)}{1-a}\right]. \tag{9}$$

The value of λ^* corresponds to the position of the boundary of the cavity.

For a concrete calculation, quartz containing water and air was selected as the multicomponent medium. The constants entering into the equation of state were selected in accordance with [5]. As the density ahead of the shock wave, there was taken the density corresponding to the external pressure ahead of the front, $p_1 = 1$ kbar.

To integrate Eq. (7), the initial value of the reduced dimensionless pressure q must be assigned, which, in accordance with (8), is equivalent to the assignment of the limiting degree of compression a. However, the value of a for rock and water with an equation of state of the form (5) is unknown; therefore, a certain degree of arbitrariness in its selection must be admitted. This is connected with the fact that the problem of a strong point explosion is posed for a medium with a previously assigned degree of compression at the shock front. With $a = 1$, we obtain the earlier problem of an explosion in an incompressible medium [1]. For the case of a one-component medium with $a < 1$, an analysis of the solution was made in [2]. For a multicomponent medium, the value of a can be regarded as the parameter of the problem, which may be selected in several ways, taking account of different mechanisms of the effect of additives of water and air on the character of explosive motions of the continuous medium.

However, we must first take note of an important property of the solutions of the differential equation (7). Figure 1 gives curves of $P(\lambda)/p_2$ calculated for quartz with zero moisture and gas saturation. Curve 1 corresponds to $a = 0.9228$, curve 2 to $a = 0.8178$, and curve 3 to $a = 0.6773$. The curves point to a considerable dependence of the solutions of Eq. (7) on the degree of compression a. With a rise in the degree of compression, i.e., with an increase in a, the dependence $P(\lambda)$ behind the shock front becomes smoother and qualitatively approaches the solution for a strong point explosion in a gas. On the other hand, curve 1, corresponding to a weak compression at the shock front, has the characteristic features of the solution of the problem of an explosion in an incompressible medium [1]. From Fig. 1 it can be seen that, at the center of symmetry, there is a cavity, expanding in accordance with a self-similar law.

Equations (6) and (7) show that moisture saturation of the medium has an effect both on the value of the degree of compression a at the front and on the behavior of the thermodynamic functions f and φ and their derivatives. In order to bring out the role of each of these factors, two ways of selecting the parameter a can be proposed. One of them consists in the fact that, for the different cases of gas and moisture saturation, exactly the same value of the saturation a is taken. Thus, effects appear, connected with taking account of the gas and moisture saturation behind the shock front and with neglect of the effect of the change in the degree of compression at the shock front itself.

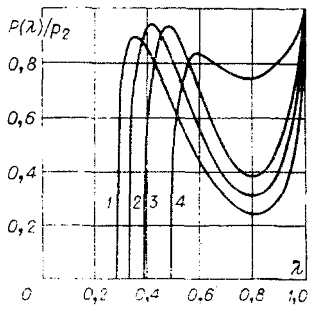

Fig. 3

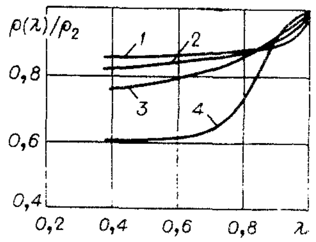

Fig. 4

Figure 2 gives two types of curves, obtained for weak ($a = 0.95$, curves 1-3) and strong compression ($a = 0.75$, curves 4-6) at the shock front. The calculated dependences 1 and 4 were obtained for quartz not containing impurities; curves 2 and 5 were obtained for quartz containing 1% water by weight; curves 3 and 6 were obtained for quartz containing $1.22 \cdot 10^{-3}$% air by weight. Curves 1-3 show that, with a small degree of compression, it is essential to take account of moisture and gas saturation in the equation of state of the medium. The effect of even small amounts of water or air leads to a situation in which the change in the pressure with the parameter λ becomes smoother. For a large compression, the effect of impurities of gas or water is less significant.

Another selection of the parameter a makes it possible to take account of the change in the degree of compression as a function of the content of water or air in the medium and, simultaneously, to take account of the multicomponent nature of the problem in the equation of state. For this purpose, the pressure p_2 was fixed at the shock wave and the parameter a was calculated using the formula $a = v(p_2)/v(p_1)$ [expression (6) was used for v(p)]. The results of solution (7) with such a selection of a are given in Figs. 3 and 4. Figure 3 illustrates the calculated dependences $P(\lambda)/p_2$ for a different content of water in quartz [1) $a = 0.8640$, $R_W = 0$; 2) $a = 0.8582$, $R_W = 0.01$; 3) $a = 0.8525$, $R_W = 0.02$; 4) $a = 0.7920$, $R_W = 0.15$]. Figure 4 gives curves of $\rho(\lambda)/\rho_2$ [1) $a = 0.8640$, $R_W = 0$; 2) $a = 0.8365$, $R_W = 0.05$; 3) $a = 0.7920$, $R_W = 0.15$; 4) $a = 0.6320$, $R_W = 1.00$]. The greater compressibility of water and air in comparison with quartz leads to an appreciable decrease in the value of a, even for a small moisture content (the addition of 1% water leads to a decrease in the value of a by approximately 10%). Therefore, with a rise in the content of water in the medium, there is a rapid decrease in a, which (physically) means a transition to the problem of an explosion in a strongly compressible medium. This can be seen directly from the similarity of Figs. 1 and 3 for small values of a. A consideration of a gas-saturated medium leads to analogous conclusions.

Analyzing the results of the calculations, the following conclusions can be drawn. With a relatively small compressibility ($a > 0.9$), the effect of moisture and gas saturation of the medium on the development of a strong point explosion is due both to a change in the thermodynamics of the medium and to a change in the compressibility at the shock front. For large degrees of compression ($a < 0.85$), the last of the above-named mechanisms becomes the principle one. Thus, taking account of gas and moisture saturation leads to a less sharp drop in the pressure behind the shock front. This can be seen directly in Figs. 2 and 3.

The dependences $\rho(\lambda)/\rho_2$, illustrated in Fig. 4, have a somewhat different character. In the case of the solid component alone, the density behind the shock front falls rapidly to its initial value and then remains practically unchanged, which is in good agreement with the known fact of the weak compressibility of the substance behind the shock front. With a rise in the moisture and gas saturation of the medium, the drop in the density becomes smoother, and the medium remains compressible far behind the shock front.

The above-described picture of the behavior of the pressure, the velocity, and the density leads to an increase in the specific energy of the medium behind the shock front, which results in a sharper damping of the shock wave itself with distance, i.e., a decrease in the peak pressures with a rise in the content of water and gas at identical distances from the point of the explosion. Mathematically, this follows from the form of the curves in Figs. 3 and 4 and formula (9), reflecting the law of conservation of the energy of the explosion E_0. In accordance with (9), with a rise in the content of water and gas, the parameter α rises sharply, and the effective energy of the explosion E falls, since, from (2), $E = E_0/\alpha$.

We note that the results obtained are in qualitative agreement with the conclusions of [6], whose authors studied the problem of the expansion of the cavity in a multicomponent medium.

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DETERMINATION OF EJECTION EXTRACTION WITH THE EXPLOSION OF AN UNDERGROUND FUSE-TYPE CHARGE IN A TWO-LAYER MEDIUM

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In [1], in a pulsed-hydrodynamic statement, an investigation was made of the problem of determining the ejection extraction with the explosion of a fuse-type discharge in a two-layer medium. This problem is solved below with more general assumptions.

Let the ground consist of two layers of identical density, differing in the values of the critical velocity. The upper layer with a thickness H is characterized by the critical velocity v_1, while the lower layer, of unbounded thickness, has the critical velocity v_2. At a depth h from the surface of the ground, there is a fuse-type charge, modeled in the vertical plane by a source with a power of 2q. It is required to determine the limit of the ejection extraction, taking account of its lines of flow, and taking $v = v_1$ behind it in the upper layer and $v = v_2$ in the lower layer, where v is the value of the velocity. We note that, in distinction from a solid–liquid model of an explosion (see, for example, [2, 3]), in the present work, as in [1], the condition $v > v_0$ (v_0 is the critical velocity) is not imposed in the region of the motion. Only such ejection schemes are considered in which the point of branching of the boundary of the ejection extraction lies below the line of separation of the layers. Depending on the ratio of the critical velocities v_1 and v_2, two variants are studied.

Variant 1. Let $v_1 < v_2$. The corresponding scheme of the ejection extraction is illustrated in Fig. 1 (by virtue of the symmetry with respect to the y axis, only the right-hand half of the ejection extraction is shown; this region is denoted by G_Z and its boundary ABMNRCDA, by Γ_Z). We note that the condition $|y_0| \geq H_0$ (y_0 is the value of y at the point B) is clearly satisfied if $h \geq H$. The starting parameters of the problem are q, h, H, v_1, and v_2.

We introduce dimensionless variables by the relationships

$$z^* = z/H, \quad w^* = w/q, \quad v^* = vH/q, \tag{1}$$

where $z = x + iy$ is the physical plane; $w(z) = \varphi + i\psi$ is the complex flow potential. The solution will then depend only on three parameters:

$$h^* = h/H, \quad v_1^* = v_1H/q, \quad v_2^* = v_2H/q,$$

since $H^* = 1$, $q^* = 1$. In what follows, for simplicity, we shall omit the superscript asterisk for the dimensionless variables.

The problem described reduces to the following boundary-value problem: Determine the unknown sections of the boundary Γ_Z of the region G_Z in such a way that the function $w(z) = \varphi(x, y) + i\psi(x, y)$, analytical in G_Z and continuous in $\overline{G}_Z$ (except for the point A), will satisfy the following conditions at Γ_Z:

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